Double Compare: Strategies for Combining and Comparing

Through the game of *Double Compare,* students develop their strategies for combining two numbers and for reasoning about quantity. The following scenes from a classroom illustrate situations that commonly arise and show how to adjust the game for students at different levels.

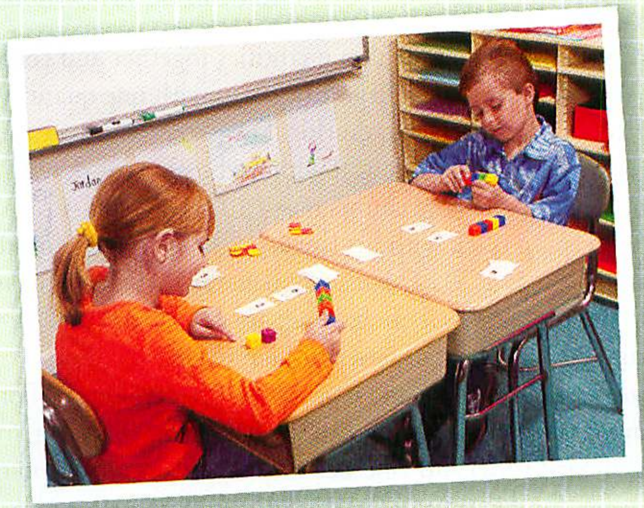
Counting Objects

As Bruce turns up 3 and 0 and Sacha turns up two 8 cards, Bruce begins by reminding himself, "Count those little things [the pictures on the cards]." Then, as Sacha watches, he counts each picture on the 3 card, touching them as he says the numbers. He announces that he has 3.

Sacha places her cards side by side, overlapping the edges. She counts slowly, touching all the pictures as she says the numbers, but she skips a few pictures, counts a few twice, and comes up with a total of 13. Bruce says that he thinks 8 and 8 is 18. Although they are aware that at least one of these totals is inaccurate, they realize that regardless, Sacha's total is greater than Bruce's total of 3, and they are ready to move on.

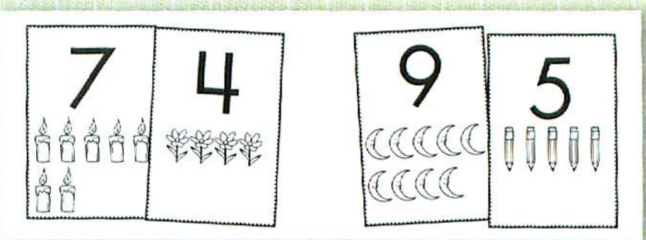
At this point the teacher steps in and asks them to recount Sacha's total, slowly. After a couple of trials, Sacha and Bruce both come up with a total of 16. The teacher suggests that they use connecting cubes to help them find the totals on their cards. Because cubes—unlike the pictures on the card—can be moved around, they can make it easier for students to keep track of what they have counted and what they have left to count.

Sacha and Bruce both need to count by ones to be sure of their totals, and counting totals greater than 10 is challenging for them. The teacher plans to return in a few minutes to see whether the cubes are helpful. If Sacha and Bruce are still having difficulty working with larger numbers, she will suggest that they play with only the 1–6 cards. Later in the session, she will call together students having difficulty and will work with them as they play *Double Compare.*



Sacha and Bruce benefit from using connecting cubes to find the totals on their cards.

Counting and Counting On



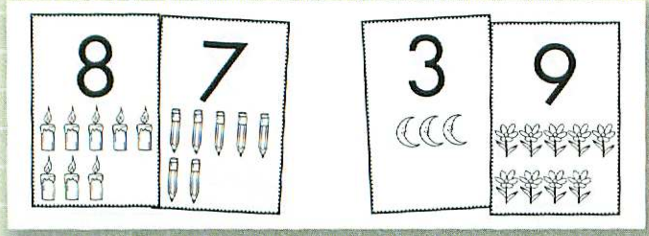
Vic's cards, Diego's cards

Vic and Diego get right to work finding their totals. Diego counts quietly to himself. He begins at 9, and then counts "10, 11, 12, 13, 14." With each number he says, Diego uses his right index finger to bend back one of the fingers on his left hand. When he has bent back all the fingers on his left hand, he stops counting and announces that he has 14.

Meanwhile, Vic is still counting. He begins by looking first at the 7 card and counting from 1 to 7. Then, he turns to the 4 card and begins counting "8, 9 . . ." When Diego announces his total of 14, Vic loses his place and begins counting again. He counts to 7, and then he counts the pictures on the 4 card, saying "8" as he points to the first picture, "9" as he points to the second, and so on, until he reaches 11. The boys agree that Diego has the greater total.

Like Sacha, Vic puts the two quantities together and counts them all, starting at 1. Diego can begin with one quantity and count on. In order to do this, Diego treats 9 as a unit; that is, he can think of it as 9 without breaking it down into 1s again. Then he counts on—"10, 11, 12, 13, 14"—while keeping track with his fingers of how many he needs to add (1, 2, 3, 4, 5). The teacher believes that the game is at an appropriate level of challenge for Diego and Vic. In future sessions, she will observe them to see how their strategies for counting and combining are developing. For example, she will note whether Vic continues counting from 1 each time, and whether they have begun developing strategies for determining particular combinations without counting.

"Just Knowing" Number Combinations



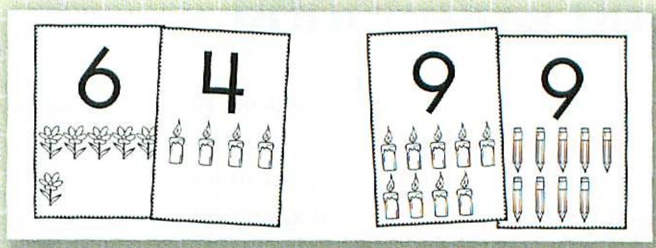
Nicky's cards, Isabel's cards

As Nicky and Isabel turn over their cards, Nicky immediately announces that she has 15 and then looks over at Isabel's cards. The teacher reminds Nicky to let Isabel find her own total. Meanwhile, Isabel counts almost inaudibly to herself "10, 11, 12," and then says that her total is 12. Nicky says, "Me! I won."

Teacher: How did you get your totals?

Nicky: Because I know 8 and 7 makes 15. Because it's easy.

Isabel: I counted in my mind.

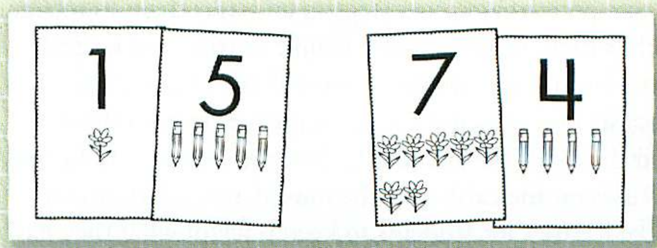


Nicky's cards, Isabel's cards

On the next round, Nicky again immediately announces her total, and then waits impatiently while Isabel slowly counts on from 9.

When the teacher again asks how the girls found their solutions, Nicky is still unable to explain. She seems either to have memorized some number combinations, or to have developed strategies for finding solutions to number combinations quickly. As Nicky is eager to play at a faster pace than she can with Isabel, the teacher decides to ask her to play with Paul, who is also finding number combinations quickly. To provide further challenge, she may ask Nicky and Paul to turn over three cards on a round.

Reasoning About Number Combinations

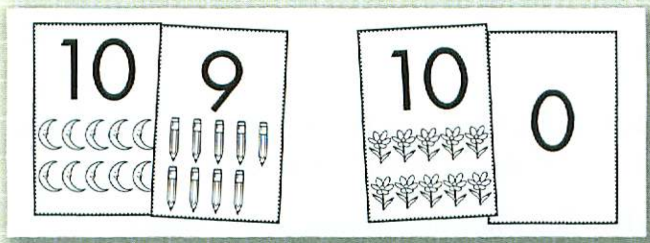


Jacob's cards, Paula's cards

Jacob: 1 and 5 is 6. I have 6.

Paula: 8, 9. [Short pause] Me! Because you have 6 and I have more. Let's do it again.

Jacob: Me! 9 is bigger than 0. You know because it's just your eyes that tell you.



Jacob's cards, Paula's cards

Paula and Jacob are reasoning about the number pairs without necessarily needing to add them up. The teacher has observed in previous sessions that Jacob and Paula are skilled at counting, comparing, and combining numbers. However, she believes that this game is deepening their understanding of numbers and number relationships as they explore ways to reason about numbers.

Evidence of Early Algebraic Thinking?

When students are able to determine who wins without actually finding the two totals, they probably have a rule in mind—a generalization about how the numbers work. For example, when Jacob had 1 and 5, and Paula had 7 and 4, they knew that Paula's total would have to be larger because 7 is *already* larger than 1 and 5. (The rule underlying their strategy could be written using algebraic notation: If $a + b < c$ and $d > 0$, then $a + b < c + d$.) When Jacob's cards were 10 and 9, and Paula's were 10 and 0, they knew that Jacob's total was higher by comparing 9 and 0. (If $x > y$, then $z + x > z + y$.) Clearly, such notation is not appropriate for first graders. What *is* appropriate is helping them learn to verbalize their ideas as they work in small groups or have whole-class discussions.

Libby: Sometimes you can tell who gets to say "me" without counting.

Teacher: Can you explain how you can tell?

Libby: If one person has two really big numbers, and the other person has two tiny numbers, then you know.

Teacher: Felipe, how did you know so quickly that Tamika's cards showed more?

Felipe: We had one round where I got 2 and 5 and Tamika got 2 and 6, and we didn't have to count. You only have to look at 5 and 6 because the 2s don't matter.

Tamika: Yeah, we both had a 2, so we just did 5 and 6 and 6 is bigger!

Edgar and Stacy played a round. Edgar's cards were 2 and 3, and Stacy's were 5 and 5. Edgar "just knew" that 5 and 5 was bigger than 2 and 3. Although getting him to verbalize his thinking was difficult, he eventually explained it.

Edgar: I have one 5 (2 + 3). She's got two 5s. 2 is more than 1.

In this struggle to verbalize the thinking behind one's strategy lies the important algebra work of the elementary grades.